MATH1013.com

We Make Math Easy.

Chapter 4.9

Tutorial Length
1 Hour 10 Mins

Homework ~ Tutorials ~ Past Tests

Important

Math 1013 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1013.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1013 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

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Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.
- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.
- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!
- Keep all of your MATH1013.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.
- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. Please see the account terms and conditions for more details.

Contact

Questions, Concerns, Comments? info@math1013.com
Please note we are unable to offer tutoring assistance over e-mail.

Definition: Antiderivative

A function $F(x)$ is called an antiderivative of $f(x)$ if $F'(x) = f(x)$

If $f(x) = 2x$, then which of the following would be considered an antiderivative?

$F(x) = x^3$
$F(x) = x^2 + 4$
$F(x) = x^2 + 2x$
$F(x) = x^2$
$F(x) = x^2 - 3$

Note that you can have an infinite number of anti-derivatives for a function by varying the constant. It is for this reason we always add a 'C' at the end of antiderivatives, where C represents any constant.

Table of Antiderivatives

Function	General Antiderivative
$kf(x)$	$kF(x) + C$
$f(x) \pm g(x)$	$F(x) \pm G(x) + C$
k	$kx + C$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$ or x^{-1}	$\ln\|x\| + C$
e^x	$e^x + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
a^x	$\frac{a^x}{lna} + C$
a^{kx}	$\frac{a^{kx}}{k\,lna} + C$

Example (VID_9818)
Find the antiderivative of $f(x) = x^3$

Example (VID_0382)
Find the antiderivative of $f(x) = 2x^3$

Example (VID_5467)
Find the antiderivative of $f(x) = 2x^3 + 3x - 4$

Example (VID_0971)
Find the antiderivative of $h(x) = \frac{4}{x} + \frac{3}{x^2} - 2 + e^{5x}$

Example (VID_5037)
Find the antiderivative of $h(x) = \sqrt[3]{x^4} - \frac{3}{\sqrt{x}} + 5^{2x}$

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How To Quickly Add 1 To A Fraction

$$\frac{a}{b} + 1 = \frac{a+b}{b}$$

Keep the denominator the same, the numerator is the sum of the numerator and denominator.
Any negative signs belong to the numerator.

Example

$\frac{5}{2} + 1$ $\frac{3}{7} + 1$ $-\frac{1}{4} + 1$ $-\frac{8}{3} + 1$

Dividing By A Fraction

When dividing by a fraction, simplify the expression by taking the reciprocal and turning the division into a multiplication

$$\frac{a}{b/c} = a\frac{c}{b} = \frac{ac}{b} \qquad \frac{a/b}{c/d} = \frac{a}{b}\frac{d}{c} = \frac{ad}{bc}$$

Example

$\frac{x}{2/3}$ $\frac{-4x}{1/2}$ $\frac{x/2}{4/3}$ $\frac{2x^{-4/5}}{-4/5}$

Trigonometric Antiderivatives [VID_0305]

Function	General Antiderivative
$\cos x$	$sinx + C$
$\sin x$	$-cos\, x + C$
$\sec^2 x$	$tanx + C$
$\sec x \tan x$	$\sec x + C$
$\dfrac{1}{1+x^2}$	$\arctan(x) + C$ $\tan^{-1}(x) + C$
$\cos(kx)$	$\dfrac{1}{k} sin(kx) + C$
$\sin(kx)$	$-\dfrac{1}{k} cos(kx) + C$
$\sec^2(kx)$	$\dfrac{1}{k} tan(kx) + C$
$\sec(kx)\tan(kx)$	$\dfrac{1}{k} sec(kx) + C$

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Example [VID_1708]

Find the antiderivative of $f(x) = 3\sin x - 2\cos x$

Example [VID_4403]

Find the antiderivative of $f(x) = \sec^2(3x) - 2x^3$

Example [VID_7399]

Find the antiderivative of $f(x) = \frac{5}{1+x^2} + \sin(-2x) - 2e^x + \pi$

Example [VID_8545]
If $f'(x) = x^2 + 2\sin(x)$, what is $f(x)$?

Example [VID_9079]
If $f''(x) = 2x + e^{-x} + 3 +$, what is $f(x)$?

Example [VID_4379]
Given $\frac{ds}{dt} = \frac{3t^4 - 2t^2 + 3}{t^4}$, find $s(t)$.

Example [VID_8958]
Given $\frac{dv}{du} = (u-1)(u+2)$, find $v(u)$.

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$\sec(kx)\tan(kx)$	$\frac{1}{k}\sec(kx) + C$
$\frac{1}{1+x^2}$	$\arctan(x) + C$ $\tan^{-1}(x) + C$

Antiderivative Application Questions

In general, when you are given the derivative of a function, $f'(x)$, and want to find the original function $f(x)$, all you have to do is find the antiderivative.

With application questions, once you find the antiderivative, you sometimes have to solve for the constant C. This occurs whenever the question gives you information about what $f(x)$ equals to for some value of x.

These types of problems are sometimes referred to as Initial Value Problems since an initial value is provided that lets you solve for C.

Example [VID_4620]

If $f'(x) = (2x+1)^2$, what is $f(x)$ if $f(1) = 2$

Example [VID_8864]

If $\frac{dh}{du} = e^{2u}(e^u + 6)$, what is $h(u)$ if $h(0) = 4$?

Example [VID_5241]

Solve the initial value problem

$y' = \sin(x) + e^x, \quad y(0) = 1$

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$\frac{1}{1+x^2}$	$\arctan(x) + C$ $\tan^{-1}(x) + C$

Finding $f(x)$ when given tangent slope $f'(x)$ and point (x_0, y_0)

Determine the function $f(x)$ whose tangent line has slope $f'(x)$ and passes through the point (x_0, y_0)

Step 1) Take the antiderivative of $f'(x)$ to find $f(x)$
Step 2) *Optional* If given a point the point must pass through, substitute in to $f(x)$ to solve for C

Example (VID_0827)

Determine the function $f(x)$ whose tangent line has slope $f'(x) = x^2 + 4$ if $f(x)$ passes through the point $(0,5)$

Example (VID_8625)

Determine the function $h(x)$ whose tangent line has slope $h'(x) = e^{3x} - 2$ if $h(x)$ passes through the point $(0,4)$

Position, Velocity, Acceleration [VID_8899]

Let $s(t)$ represent the position of an object at any point in time

The rate of change of position is velocity: $s'(t) = v(t)$
The rate of change of velocity is acceleration: $v'(t) = a(t)$

To work your way backwards from $a(t)$ to $v(t)$, you find the antiderivative of $a(t)$
To work your way backwards from $v(t)$ to $s(t)$, you find the antiderivative of $v(t)$

Example [VID_8899] A particle moves with acceleration $a(t) = 1 + 4t - 2t^2$. It's initial velocity is $v(0) = 3m/s$ and it's initial displacement is $s(0) = 10$m. Find the position after t-seconds.