MATH1013.com

We Make Math Easy.

Chapter 3.1

Tutorial Length
1 Hour

Homework ~ Tutorials ~ Past Tests

Important

Math 1013 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1013.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1013 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

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Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.
- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.
- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!
- Keep all of your MATH1013.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.
- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. Please see the account terms and conditions for more details.

Contact

Questions, Concerns, Comments? info@math1013.com
Please note we are unable to offer tutoring assistance over e-mail.

The Derivative

What does the derivative represent?

The derivative represents the **slope of a tangent line** at a given point.
The derivative can also be thought of as the **instantaneous rate of change**.

Formula

The following formula is known as the **limit definition of the derivative**.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative Notations

$$y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) \quad \text{"Derivative with respect to } x\text{"}$$

All of these notations are equivalent!

$\frac{d}{dx}$ is the operator to perform a derivative, much like + is addition

Derivatives - Constant Rule

Derivative Of A Constant

$if \;\; f(x) = c \;\; then \;\; f'(x) = 0$

$$\frac{d}{dx}(c) = 0$$

This rule tells us the derivative of any constant will always be 0.

Examples (VID_7417)

Given $y = 3$, find $\frac{dy}{dx}$

Given $y = -\frac{2}{5}$, find y'

Given $f(x) = 97\pi$, find $f'(x)$

Given $f(x) = e^2$, find $\frac{d}{dx}f(x)$

Determine $\frac{d}{dx}(1)$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0

Derivatives - Power Rule

Derivative Of A Power Function

$if \;\; f(x) = x^n \quad then \;\; f'(x) = nx^{n-1}$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Bring the exponent down in front, and reduce the exponent by 1.

Example (VID_0954) Given $y = x^3$, find $\frac{dy}{dx}$

Example (VID_3292) Given $y = x^{-4}$, find y'

Example (VID_8083) Given $y = x$, find y'

Example (VID_4623) Given $f(x) = x^{\frac{3}{2}}$, find $f'(x)$

Example (VID_1676) Given $f(x) = \sqrt[4]{x^5}$, find $\frac{d}{dx}f(x)$

Example (VID_2823) Determine $\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}

Derivatives - Constant Times A Function

Derivative Of Constant Times A Function

$$if \;\; g(x) = c \cdot f(x) \;\; then \;\; g'(x) = c \cdot f'(x)$$

$$\frac{d}{dx}\big(c \cdot f(x)\big) = c \cdot \frac{d}{dx}\big(f(x)\big)$$

Keep the constant and multiply by the derivative of $f(x)$.

Example (VID_5015) Given $y = -5x$, find $\frac{dy}{dx}$

Example (VID_3870) Given $y = 2x^3$, find $\frac{dy}{dx}$

Example (VID_4415) Given $y = \frac{x^{-4}}{5}$, find y'

Example (VID_2062) Given $s(t) = -2t^{\frac{3}{2}}$, find $s'(t)$

Example (VID_2486) Given $f(x) = \frac{5}{\sqrt{x}}$, find $\frac{d}{dx}f(x)$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
cx	c
cx^n	cnx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$

Derivatives - Sum/Difference

Derivative Of Sum/Difference Of Functions

$if\ \ f(x) = u(x) + v(x) \quad f'(x) = u'(x) + v'(x)$
$if\ \ f(x) = u(x) - v(x) \quad f'(x) = u'(x) - v'(x)$

$$\frac{d}{dx}\big(u(x) + v(x)\big) = \frac{d}{dx}\big(u(x)\big) + \frac{d}{dx}\big(v(x)\big)$$
$$\frac{d}{dx}\big(u(x) - v(x)\big) = \frac{d}{dx}\big(u(x)\big) - \frac{d}{dx}\big(v(x)\big)$$

Find the derivative of each term and add/subtract them.

Example (VID_6214) Given $y = x^3 - x^2 + 3$, find $\frac{dy}{dx}$

Example (VID_9720) Given $y = 2x^3 - 4x + 2$, find y'

Example (VID_3061) Given $s(t) = \sqrt{t^3} - \frac{2}{t} + 3t + \frac{3t^2}{4} - 2$, find $s'(t)$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$
$u(x) \pm v(x)$	$u'(x) \pm v'(x)$

Derivatives - Sum/Difference

Example (VID_2994) Find y' if $y = -2x^{-4} + \frac{3}{x} - \pi x + x^2 + e$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$
$u(x) \pm v(x)$	$u'(x) \pm v'(x)$

Example (VID_2197) Find $\frac{dy}{dx}$ if $y = \frac{x^2+2\sqrt{x}-3}{x}$

Example (VID_0380)

Assuming a, b, and c are constants, differentiate the following.

$y = ax^2 + bx + c$

Example (VID_4237)

Assuming a, b, and c are constants, differentiate the following.

$y = x^a + \frac{b}{x} + cb$

Derivatives - Exponential Functions

Derivative Of An Exponential Function [VID_7152]

$if\ \ f(x) = a^x \ \rightarrow f'(x) = a^x \ln(a)$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

When a = e

$if\ \ f(x) = e^x \quad f'(x) = e^x$

$$\frac{d}{dx}(e^x) = e^x$$

Example [VID_0444]

Find $\frac{dy}{dx}$ if $y = 3e^x - 4^x + 7$

Example [VID_0171]

Differentiate the function $y = e^x + x^e$

Example [VID_0733]

Differentiate the function $y = 3x^4 - \pi^x + 6e^x - 2$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$
$u(x) \pm v(x)$	$u'(x) \pm v'(x)$
e^x	e^x
a^x	$a^x \ln(a)$

Note

Technically the textbook introduces the trig integrals a few sections later, but they are simple enough to be learned now. The tutorials will be shown again in the appropriate section later on (Section 3.3)

Derivative Of Trig Functions [VID_3648]

$if\ \ f(x) = \sin x \ \rightarrow\ \ f'(x) = \cos x$
$if\ \ f(x) = \cos x \ \rightarrow\ \ f'(x) = -\sin x$
$if\ \ f(x) = \tan x \ \rightarrow\ \ f'(x) = \sec^2 x$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Example [VID_8564]

Find $\frac{dy}{dx}$ if $y = 3\sin x - 2\cos x + tan\, x$

Derivative Notations

y' $\quad f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}f(x)$

List Of Rules

Function	Derivative
c	0
x	1
x^n	nx^{n-1}
$c \cdot f(x)$	$c \cdot f'(x)$
$u(x) \pm v(x)$	$u'(x) \pm v'(x)$
e^x	e^x
a^x	$a^x \ln(a)$
$sin\, x$	$cos\, x$
$cos\, x$	$-sin\, x$
$tan\, x$	$sec^2\, x$

Example [VID_1980]

Differentiate the function $f(x) = \sin(x) - 4e^x + 3x^2 - 1$

Example [VID_0600]

Calculate $s'\ if\ s = 3t^2 + 2e^t - 2\cos t - \pi\, tan\, t$

What Is A Higher Derivative

Given a function $f(x)$, the derivative is referred to as $f'(x)$
The derivative of $f'(x)$ is then referred as $f''(x)$, this is called the second derivative.
The derivative of $f''(x)$ is then referred to as $f'''(x)$, this is called the third derviative.

First Derivative Notations

$$y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx} = D_x[f(x)] \quad "Derivative\ with\ respect\ to\ x"$$

Second Derivative Notations

$$y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d^2f}{dx^2} = D_x^2[f(x)] \quad "Derivative\ with\ respect\ to\ x"$$

Third Derivative Notations

$$y''' = f'''(x) = \frac{d^3y}{dx^3} = \frac{d^3f}{dx^3} = D_x^3[f(x)]$$ Derivative with respect to x

Fourth Derivative and Higher $(n \geq 4)$

$$f^{n}(x) = \frac{d^ny}{dx^n} = \frac{d^nf}{dx^n} = D_x^n[f(x)]$$ Derivative with respect to x

Example (VID_7040)

Find the 5th derivative of the function $f(x) = 2x^5 - 3x^2 + 4x + 5$

Example [VID_6886]
Find the 2nd derivative of the function $f(x) = 4e^x + 2^x + x^4 + 3x$

Example [VID_3568]
Find the 3rd derivative of the function $f(x) = 2\sin(x) + 3\cos(x)$