MATH1013.com

We Make Math Easy.

Chapter 1.3

Tutorial Length
2 Hrs 20 Mins

Homework ~ Tutorials ~ Past Tests

Important

Math 1013 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1013.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1013 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

MATH1013.com

Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.
- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.
- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!
- Keep all of your MATH1013.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.
- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. Please see the account terms and conditions for more details.

Contact

Questions, Concerns, Comments? info@math1013.com
Please note we are unable to offer tutoring assistance over e-mail.

Transformations of Functions

A function $f(x)$ can be transformed by moving the graph up or down, left or right, stretching/compressing vertically or horizontally, and reflecting vertically or horizontally in order to create a new function.

Any function can be transformed using the parameters a, b, h and k:

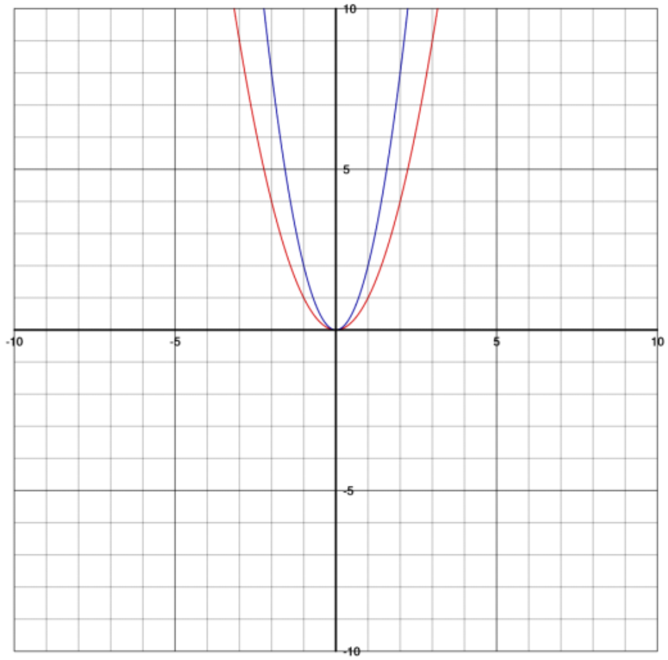
Summary

	move graph up move graph down
	move graph left move graph right
	stretch in y direction 1 compress in y direction Additionally, if a<0 reflect about the x-axis
	compress in x direction 1 stretch in x direction Additionally, if b < 0 reflect about the y-axis

<u>Order: Reflect, Stretch/Compress, Shift</u>

Parameter a: Vertical scale change all the y-values get multiplied by 'a' where as the x-values remain unchanged. If 'a' is negative, the function is reflected about the x-axis.

Scenario: $|a| > 1$ (Vertical Strech)



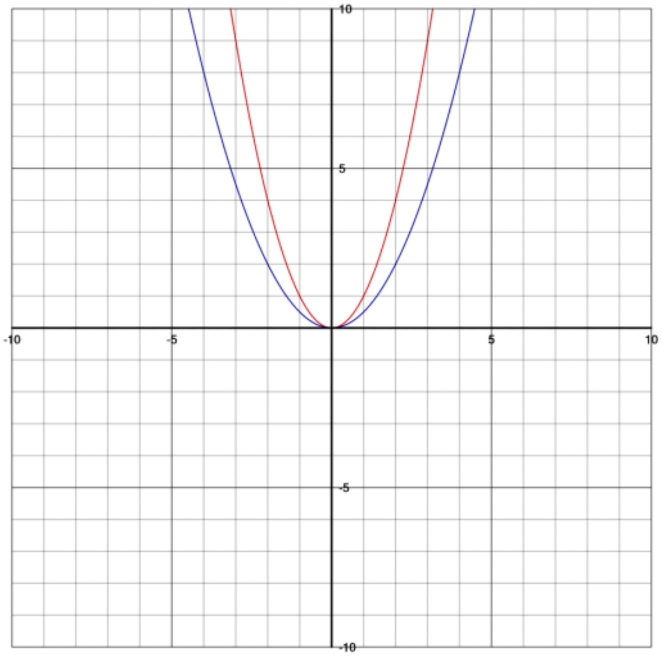
$y = x^2$
$y = 2x^2$
a=2, vertical stretch by factor of 2

Summary
$y = f(x) \quad \rightarrow \quad y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x + h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter a: Vertical scale change all the y-values get multiplied by 'a' where as the x-values remain unchanged. If 'a' is negative, the function is reflected about the x-axis.

Scenario: 0< $|a| < 1$ (Vertical Compression)



$y = x^2$

$y = \left(\frac{1}{2}\right) x^2$

a=1/2, vertical compression by factor of 2

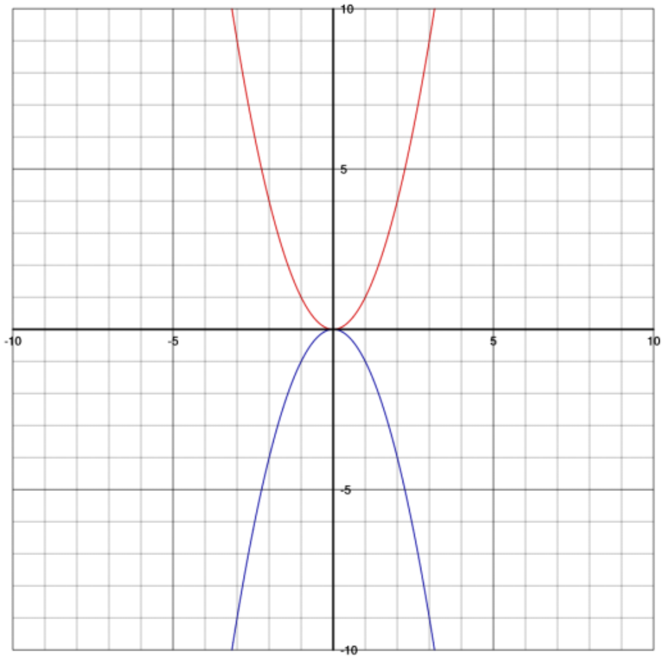
Summary

$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\lvert a\rvert > 1$ stretch in y direction $0 < \lvert a\rvert < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\lvert b\rvert > 1$ compress in x direction $0 < \lvert b\rvert < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter a: Vertical scale change all the y-values get multiplied by ‘a’ where as the x-values remain unchanged. If ‘a’ is negative, the function is reflected about the x-axis.

Scenario: $a < 0$ (Reflection about x-axis)



$y = x^2$
$y = -x^2$
a= -1, Reflection about x-axis

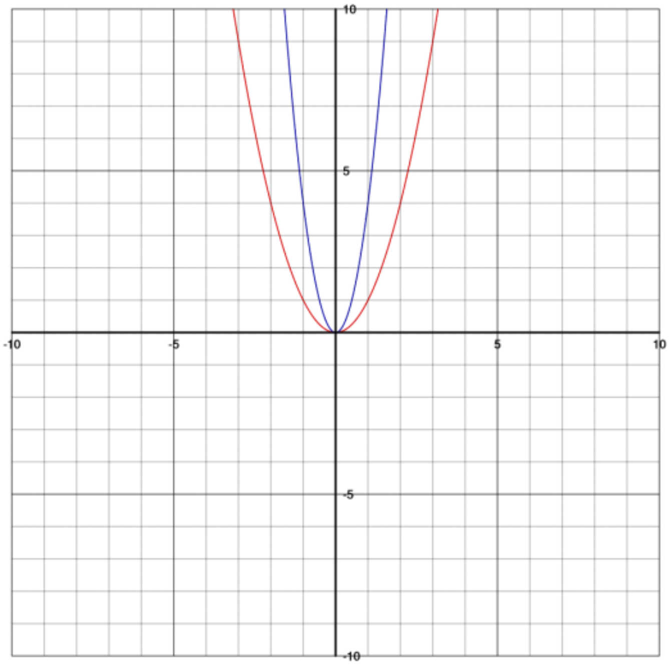
Summary

$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter b: Horizontal scale change all the x-values get multiplied by 'b' where as the y-values remain unchanged. If 'b' is negative, the function is reflected about the y-axis

Scenario: $|b| > 1$ (Horizontal Compression)



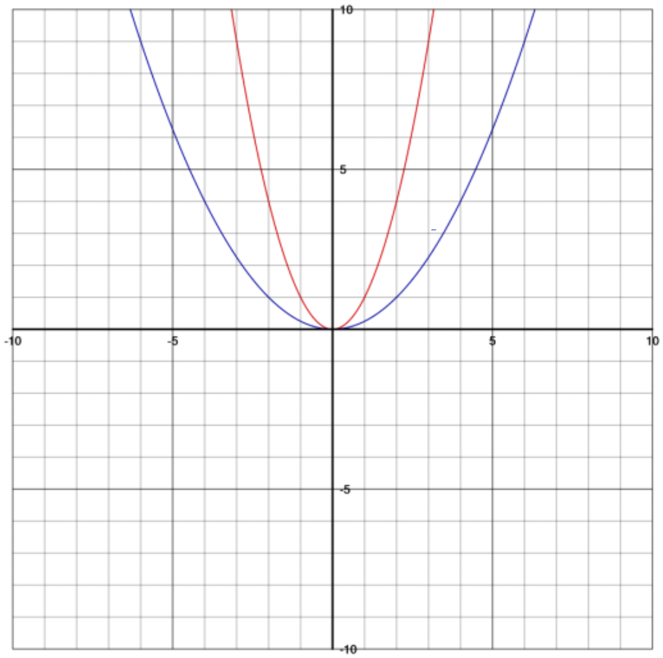
$y = x^2$
$y = (2x)^2$
b=2, horizontal compression by factor of 2

Summary
$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter b: Horizontal scale change all the x-values get multiplied by 'b' where as the y-values remain unchanged. If 'b' is negative, the function is reflected about the y-axis

Scenario: 0< $|b| < 1$ (Horizontal Stretch)



$y = x^2$

$y = (\frac{1}{2}x)^2$

b=1/2, horizontal stretch by factor of 2

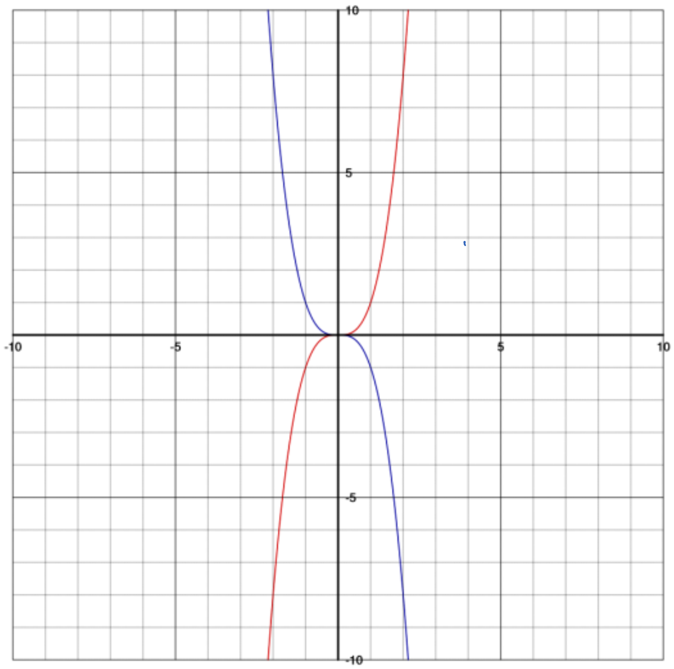
Summary

$y = f(x) \quad \rightarrow \quad y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x + h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\lvert a\rvert > 1$ stretch in y direction $0 < \lvert a\rvert < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\lvert b\rvert > 1$ compress in x direction $0 < \lvert b\rvert < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter b: Horizontal scale change all the x-values get multiplied by ‘b’ where as the y-values remain unchanged. If ‘b’ is negative, the function is reflected about the y-axis

Scenario: $b < 0$ (Reflection about x-axis)



$y = x^3$
$y = (-x)^3$
b= -1, Reflection about y-axis

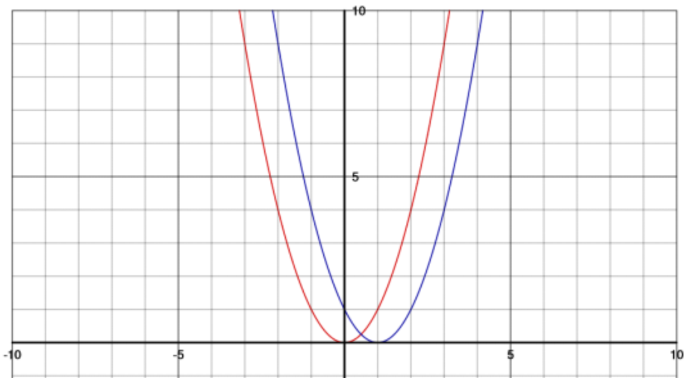
Summary
$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\lvert a\rvert > 1$ stretch in y direction $0 < \lvert a\rvert < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\lvert b\rvert > 1$ compress in x direction $0 < \lvert b\rvert < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

For illustrative purposes, we didn't use $y = x^2$ and $y = (-x)^2$ because they are the same function when simplified, so it would be difficult to show the reflection.

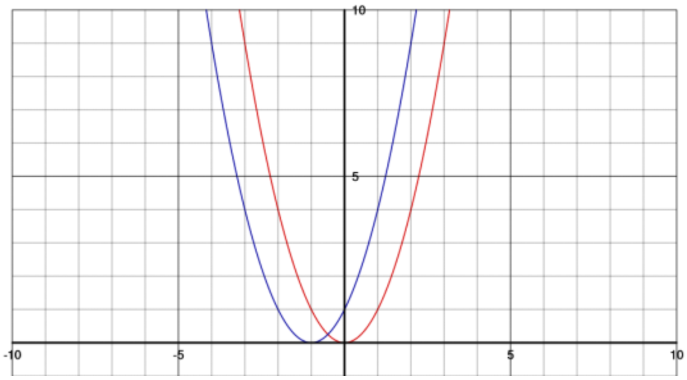
Parameter h: Horizontal translation, the function shifts left/right according to the value of h.

Scenario: $h < 0$ (shift to the right)



$y = x^2$
$y = (x-1)^2$
h= -1, shift to the right by 1 unit

Scenario: $h > 0$ (shift to the left)



$y = x^2$
$y = (x+1)^2$
h = 1, shift to the left by 1 unit

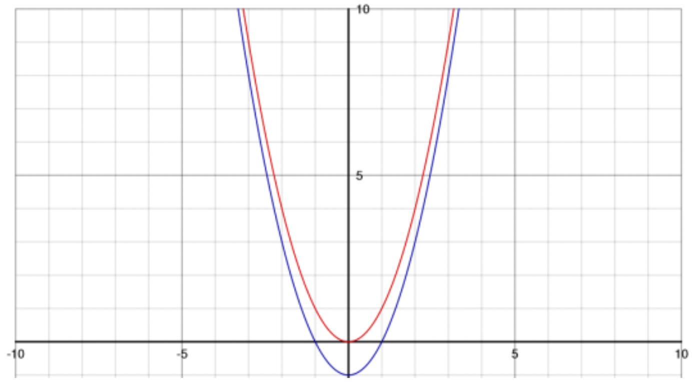
Summary

$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Parameter k: Vertical translation, the function shifts up/down according to the value of k.

Scenario: $k < 0$ (shift down)

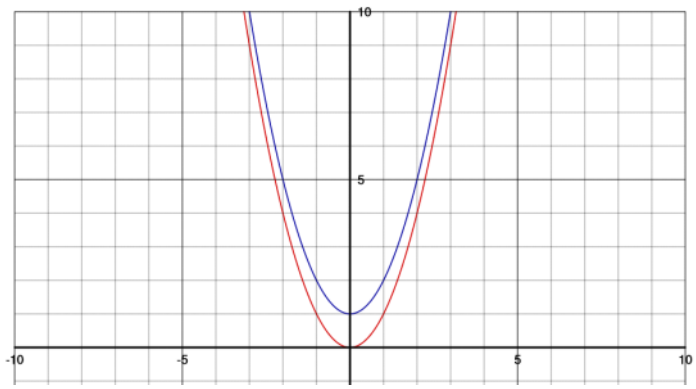


$y = x^2$
$y = x^2 - 1$
k= -1, shift down by 1 unit

Summary
$y = f(x) \rightarrow y = af[b(x+h)] + k$

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Scenario: $k > 0$ (shift up)



$y = x^2$
$y = x^2 + 1$
k= 1, shift up by 1 unit

Example Summary

$y = x^2$	
$y = 2x^2$	vertical stretch by factor of 2
$y = \left(\frac{1}{2}\right)x^2$	vertical compression by factor of 2
$y = -x^2$	reflection about x-axis
$y = (2x)^2$	horizontal compression by factor of 2
$y = (\frac{1}{2}x)^2$	horizontal stretch by factor of 2
$y = (-x)^2$	reflection about y-axis
$y = (x-1)^2$	shift to the right by 1 unit
$y = (x+1)^2$	shift to the left by 1 unit
$y = x^2 - 1$	shift down by 1 unit
$y = x^2 + 1$	shift up by 1 unit

Recall
Transforming functions
Any function can be transformed using the parameters a, b, h and k:

$y = f(x) \; y = af[b(x+h)] + k$

Summary

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

Summary

Any function can be transformed using the parameters a, b, h and k:

$$y = f(x) \quad \rightarrow \quad y = af[b(x+h)] + k$$

- b and h modify x −values
- a and k modify y −values

- If a or b is negative, it is a reflection

- When adding or subtracting (h and k values), it is a <u>shift</u>/<u>translation</u>

- When multiplying (a and b values), its is a <u>stretch</u> or <u>compression.</u>

- Horizontal movement is the opposite of what you would expect compared to similar parameters for vertical movement.

Parameter Summary

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x+h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

<u>**Order: Reflect, Stretch/Compress, Shift**</u>

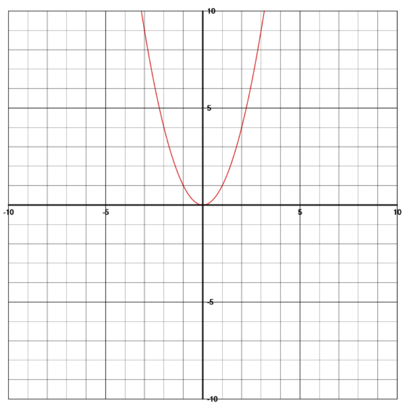
Example (VID_7052)

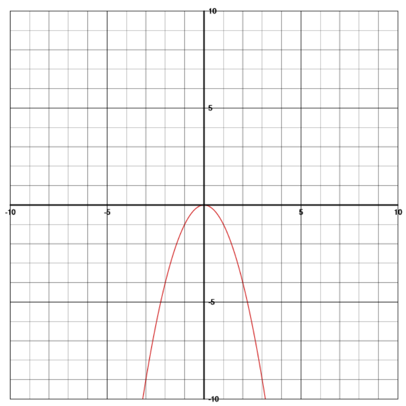
A) Describe the transformation $f(x) = x^2$ in order to get the function $f(x) = -\frac{1}{2}(x-3)^2 + 1$

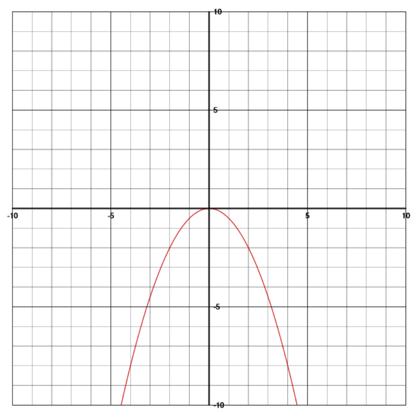
$f(x) = af[b(x+h)] + k$

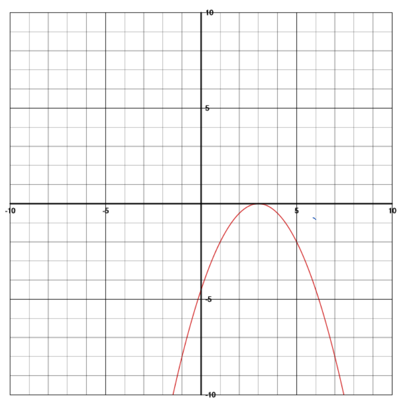
1. Reflect about x-axis to get $-x^2$
2. Compress vertically by a factor of 2 to get $-\frac{1}{2}x^2$
3. Shift to the right by 3 to get $-\frac{1}{2}(x-3)^2$
4. Shift up by 1 to get $-\frac{1}{2}(x-3)^2+1$

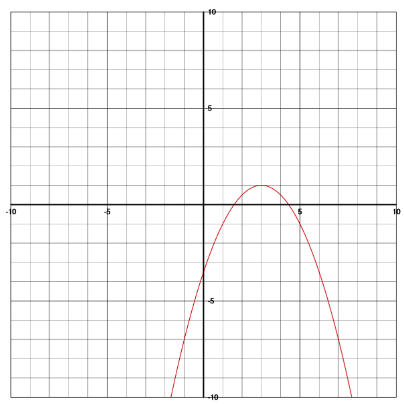
B) Plot $-\frac{1}{2}(x-3)^2 + 1$











$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y = -x^2$

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

$y = -\frac{1}{2}x^2$

x	y
-2	-2
-1	-1/2
0	0
1	-1/2
2	-2

$y = -\frac{1}{2}(x-3)^2$

x	y
1	-2
2	-1/2
3	0
4	-1/2
5	-2

$y = -\frac{1}{2}(x-3)^2 +1$

x	y
1	-1
2	1/2
3	1
4	1/2
5	-1

Example (VID_2889)

A) Describe the transformation to $f(x) = \sqrt{x}$ in order to get the function $f(x) = \sqrt{-2x-4} + 1$

Recall: $f(x) = af[b(x+h)] + k$

B) Plot $f(x) = \sqrt{-2x-4} + 1$

Example (VID_1040)

Assume the following transformations need to occur to $f(x)$

-Horizontal shift to the right by 1

-Reflection about the x-axis

-Vertical stretch by a factor of 2

-Horizontal compression by a factor of 3

A) If $f(x) = \sqrt{x}$, what is the equation of the transformed function? Recall: $y = af[b(x+h)] + k$

B) If $f(x) = x^2$, what is the equation of the transformed function? Recall: $y = af[b(x+h)] + k$

Example (VID_9308)

A) Describe the transformation of $f(x) = e^x$ in order to get the function $g(x) = 5e^{-x+2} - 1$
B) Plot $g(x)$

Recall: $y = af[b(x+h)] + k$

Example (VID_6010)

Describe the transformation of $f(x) = \cos(x)$ in order to get $g(x) = -\frac{1}{4}\cos(2x+\pi) + 3$
Recall: $y = af[b(x+h)] + k$

Example [VID_4663]

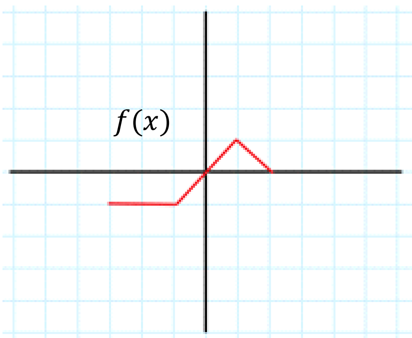
What transformations must be applied to $f(x)$, in order to get the function $g(x) = \frac{1}{2}f(-4x+8) - \frac{3}{2}$

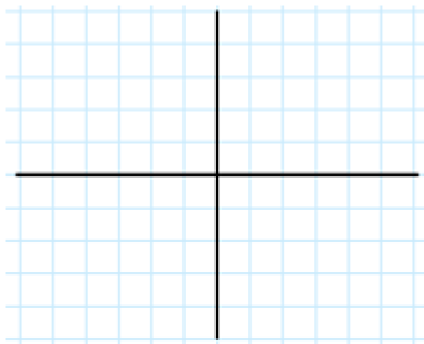
Recall: $y = af[b(x+h)] + k$

Example [VID_3401]

Given the following sketch for $f(x)$, sketch $g(x) = -f(-x-1) + 2$

Recall: $y = af[b(x+h)] + k$





Combinations of Functions [VID_3649]

Sum of functions $(f+g)(x) = f(x) + g(x)$

Difference of functions $(f-g)(x) = f(x) - g(x)$

Product of functions $(fg)(x) = f(x) \cdot g(x)$

Quotient of functions $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0$

Note 1: Sometimes the "(x)" is not shown when we are asked to find a combination of functions. For example, a question may simply give us $f(x)$ and $g(x)$ and then tell us to find $f+g$ instead of $(f+g)(x)$

Note 2: To find the domain of a combination of functions, first find the domain of each individual function and then find what is common between the domains to find the overall domain. Furthermore, when doing a quotient of functions, we must also make sure the denominator does not equal 0 when determining the domain.

Example (VID_4321]

Given $f(x) = 4x$ and $g(x) = 3x^2 - x$, find the following combination of functions as and state their associated domains.

$a)\ (f+g)(x)$

$b)\ (f-g)(x)$

$c)\ (fg)(x)$

$d)\ \left(\frac{f}{g}\right)(x)$

Example [VID_7178]

Given $f(x) = \frac{1}{\mathrm{x}}$ and $g(x) = \sqrt{x-2}$, find the following combination of functions as and state their associated domains.

$a)\ f + g$

$b)\ f - g$

$c)\ fg$

$d)\dfrac{f}{g}$

Composite Functions

$(f \circ g)(x) = f[g(x)]$ Read as "f of g of x"

"Plug x into g(x), and g(x) into f(x)" $x \to g(x) \to f\big(g(x)\big)$

Note: x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

Composite Functions

$(f \circ g)(x) = f[g(x)]$ Read as "f of g of x"

"Plug x into g(x), and g(x) into f(x)" $x \to g(x) \to f(g(x))$

Note: x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

Example

Given $f(x) = x^3 + 2x + 1$ and $g(x) = \sqrt{x+3}$

A) Find $(f \circ g)(x)$ **(VID_9143)**

B) Find $(f \circ f)(x)$ **(VID_1238)**

C) Find $(g \circ f)(x)$ **(VID_3030)**

D) Find $(g \circ g)(x)$ **(VID_7241)**

Composite Functions

$(f \circ g)(x) = f[g(x)]$ Read as "f of g of x"

"Plug x into g(x), and g(x) into f(x)" $x \to g(x) \to f\big(g(x)\big)$

Note: x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

Example
Given $f(x) = e^{3x}$ and $g(x) = \ln(x)$

A) Find $(f \circ g)(x)$ **(VID_0364 For Part A & B)**

B) Find $(f \circ g)(0)$

C) Find $(g \circ f)(x)$ **(VID_5517 For Part C & D)**

D) Find $(g \circ f)(1)$

Composite Functions

$(f \circ g)(x) = f[g(x)]$ Read as "f of g of x"

"Plug x into g(x), and g(x) into f(x)" $x \rightarrow g(x) \rightarrow f\big(g(x)\big)$

Note: x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

Example

Given $f(x) = \ln(\mathrm{x})$ and $g(x) = \mathrm{x}^2 + 2$ $h(x) = \sqrt{x+3}$

A) Find $(f \circ g \circ h)(x)$ **(VID_9770)**

A) Find $(g \circ h \circ f)(x)$ **(VID_2351)**

Composite Functions

$(f \circ g)(x) = f[g(x)]$ Read as "f of g of x"

"Plug x into g(x), and g(x) into f(x)" $x \rightarrow g(x) \rightarrow f\big(g(x)\big)$

Note: x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

Example (VID_8153)

Write $f(x)$ as a composition of 2 functions $g(x)$ and $h(x)$ such that $f(x) = (g \circ h)(x)$

$f(x) = \sqrt{x+1}$

Example (VID_4935)

Write $f(x)$ as a composition of 2 functions $g(x)$ and $h(x)$ such that $f(x) = (g \circ h)(x)$

$f(x) = 2(x+3)^2 - 3(x+3)$

Example (VID_3784)

Write $f(x)$ as a composition of 3 functions $u(x)$, $v(x)$, $w(x)$ such that $f(x) = (u \circ v \circ w)(x)$

$f(x) = \sqrt{\ln(x^2+1)}$

Domain of Composite Functions [VID_4918]

$(f \circ g)(x) = f[g(x)]$
"Plug *x* into *g(x),* and *g(x)* into *f(x)*" $x \to g(x) \to f\big(g(x)\big)$
x has to be in the domain of $g(x)$, and $g(x)$ has to be in the domain of $f(x)$

To find the overall domain of a composite function, first find the composite function and it's corresponding domain, but then also find the domain of the inner function $g(x)$. Compare the domains and find the overall domain of the composite function by seeing what is common between both.

Another way to think of it, is find the overlapping domain of $f[g(x)]$ and $g(x)$ in order to find the domain of the composite function $(f \circ g)(x)$.

Example [VID_5860]
Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{x+2}{2x-3}$, determine $(f \circ g)(x)$ and it's corresponding domain

Example [VID_1323]
Given $f(x) = e^{3x}$ and $g(x) = \ln(x)$, determine $(f \circ g)(x)$ and it's corresponding domain

Example [VID_5485]

Given $f(x) = \frac{x-2}{x+1}$ and $g(x) = sin(5x)$, find the following composite functions and the associated domain.

$a)\ (g \circ g)(x)$

$b)\ \ (g \circ f)(x)$

$c)\ (f \circ f)(x)$

$d)\ (f \circ g)(x)$