MATH1013.com

We Make Math Easy.

Chapter 1.2

Tutorial Length
50 Mins

Homework ~ Tutorials ~ Past Tests

Important

Math 1013 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1013.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1013 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

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Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.
- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.
- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!
- Keep all of your MATH1013.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.
- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. Please see the account terms and conditions for more details.

Contact

Questions, Concerns, Comments? info@math1013.com
Please note we are unable to offer tutoring assistance over e-mail.

What is a polynomial?

In general, a polynomial is represented in the following way:

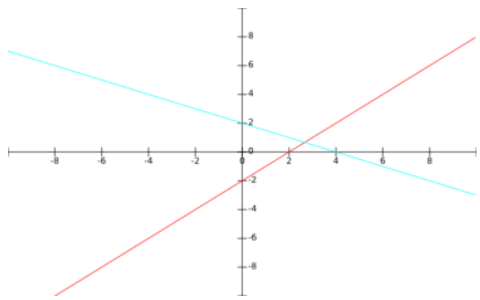
The c_n, c_{n-1}, c_2, c_1 terms are known as coefficients
The c_0 value is the constant
The 'x' term is the variable
The highest exponent, 'n' is known as the degree of the polynomial

-All variables must have whole number exponents (0, 1, 2, 3, …)
-All variables must be in the numerator
-The coefficients can be any value

Examples

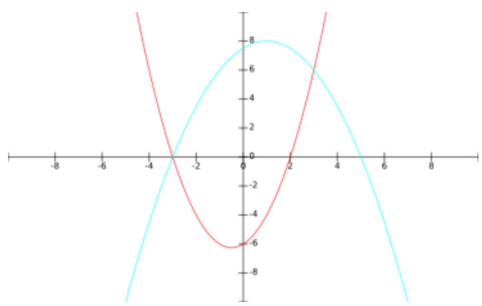
Special Polynomials

Linear Equations
Polynomial of Degree 1



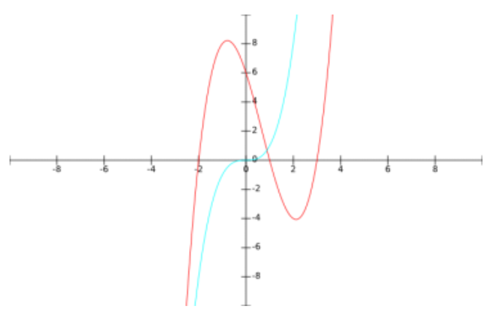
Examples

Quadratic Equations
Polynomial of Degree 2



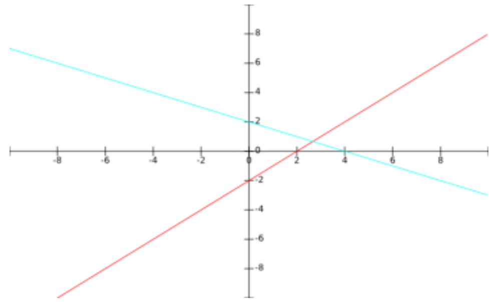
Examples

Cubic Equations - Polynomial of Degree 3



Examples

Linear Equations



Linear Equations

Polynomial of Degree 1
$P(x) \doteq c_1 x + c_0$

More commonly written as $y = mx + b$
Where m is the slope and b is the $y - intercept$

Slope Of A Line

$'m'$ is commonly used to refer to the slope of a line. The slope is the change in y-values divided by the corresponding change in x-values ("rise over run").

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Interpreting the slope:

$m > 0$ - A positive value for slope means the line is going up from left to right
$m < 0$ -A negative value for slope means the line is going down from left to right
$m = 0$ -A slope of 0 means the line is horizontal
$m = ?$ - An undefined slope means the line is vertical

Example (VID_6871)

Find the slope of the line $y = \frac{5}{3}x + 2$

Example (VID_0993)

Find the slope of the line $2x - 3y + 5 = 0$

Example (VID_2859)

Find the slope of the line passing through $(0, -2)$ and $(3,5)$

Example (VID_8933)

Find the slope of the line passing through $(-3,1)$ and $(5, -2)$

Linear Equations

3 ways to express the equation of a line:

1) Point-Slope Form: $y - y_1 = m(x - x_1)$
   m is the slope and (x_1, y_1) is any point the line passes through

2) Slope-Intercept Form: $y = mx + b$
   m is the slope and b is the y-intercept

3) Standard Form: $Ax + By + C = 0$
   A is generally a positive number

Recall, the slope of a line passing through two points (x_1, y_1) and (x_2, y_2) can be calculated by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

All 3 equations are equivalent, just a different way of expressing the line. We start with equation number 1, and then convert it to equation 2 or 3 depending on what the question is asking for.

Note: In order to form the equation of a line, you need either 2 points, or 1 point and the slope. If you have 2 points, you can find the slope using the formula.

Example (VID_7723)

What is the equation of a line passing through $(2,0)$ and $(-6,4)$? Express the answer in both slope-intercept and standard form.

Example (VID_8973)

What is the equation of a line passing through $(4,2)$ with slope $-\frac{1}{2}$? Express the answer in both slope-intercept and standard form.

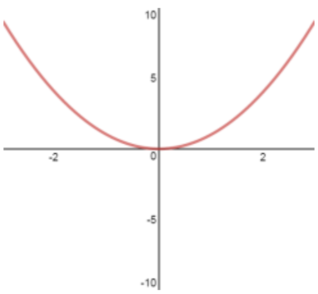
Power Function [VID_8812]

$f(x) = x^p$

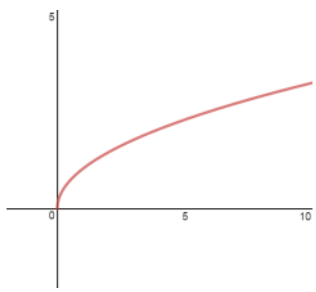
p can be any real number.
In a power function, the base is a variable and the exponent is a number.

- If p is 1 then $f(x) = x$ which is a line
- If p is even and positive integer, then it will look like similar to the parabola of x^2 [Graph 1]
- If p is odd and positive integer, then it will look like similar to the cubic function x^3 [Graph 2]
- If p is $1/n$, where n is a positive integer, then we call it a 'root' function.
  - If n is even, $f(x)$ will look like the square root function [Graph 3]
  - If n is odd, $f(x)$ will look like the cube root function [Graph 4]
  - A power function can be written in root form using the following conversion:
    $x^{\frac{1}{n}} = \sqrt[n]{x}$
- If p is -1, $f(x) = x^{-1}$, this is called a reciprocal. Note: This is equivalent to $f(x) = \frac{1}{x}$
  - If p is negative & odd, it will look similar to $f(x) = x^{-1} = \frac{1}{x}$ [Graph 5]
    If p is negative & odd, it will look similar to $f(x) = x^{-2} = \frac{1}{x^2}$ [Graph6]
- Note: Power functions can be polynomials if the exponent is a positive integer.
  For example, $f(x) = x^2$ is both a polynomial and a power function.

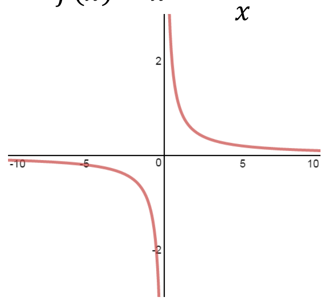
Graph 1
$f(x) = x^2$



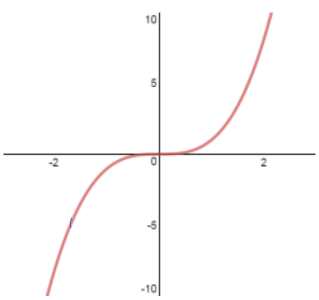
Graph 3
$f(x) = x^{1/2} = \sqrt{x}$



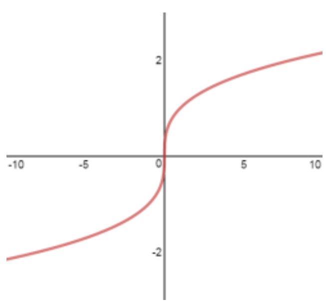
Graph 5
$f(x) = x^{-1} = \frac{1}{x}$



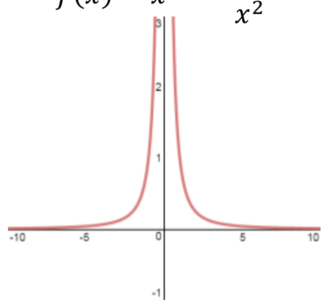
Graph 2
$f(x) = x^3$



Graph 4
$f(x) = x^{1/3} = \sqrt[3]{x}$



Graph 6
$f(x) = x^{-2} = \frac{1}{x^2}$



Rational Function [VID_0609]

A rational function is the quotient or ratio of two polynomials.

$f(x) = \frac{P(x)}{Q(x)}$ Rational functions have the restriction $Q(x) \neq 0$

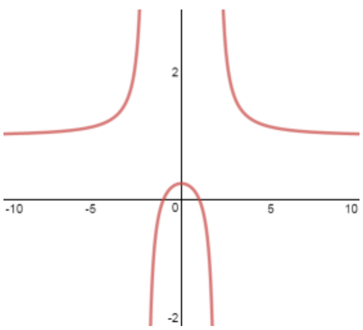
Note: A rational function can also be a power function.
Example: $f(x) = \frac{1}{x}$ is a rational function, equivalent to $f(x) = x^{-1}$ which is a power function

Examples of Rational Functions

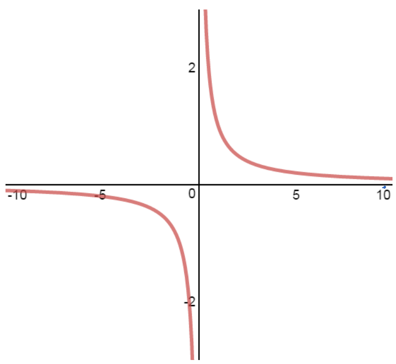
$f(x) = \frac{3x^2}{x-2}$ $f(x) = \frac{4x^2 - 2x + 1}{x^2 - 4x + 5}$ $f(x) = \frac{5}{x+1}$ $f(x) = \frac{-3x^3 + 2x}{x^2 + 3}$

Graphs of Rational Functions

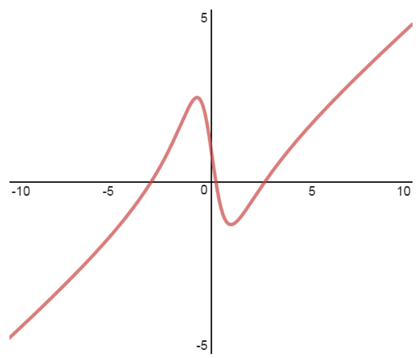
$f(x) = \frac{x^2 - 1}{x^2 - 4}$



$f(x) = x^{-1} = \frac{1}{x}$



$f(x) = \frac{x^3 - 8x + 2}{2x^2 + 2}$



Algebraic Functions [VID_7261]

A function that can be created by addition/subtraction/multiplication/division/roots starting with a polynomial function.

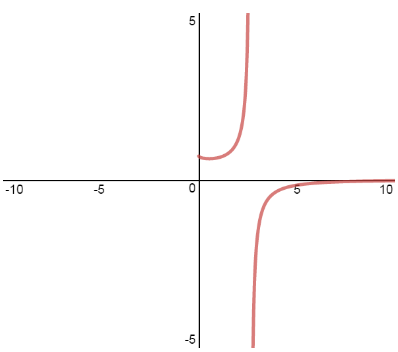
Examples:

$$f(x) = \frac{x^2 - 1}{x^2 - 4} \qquad f(x) = \sqrt[5]{x^2 - 2x} - 5x^2$$

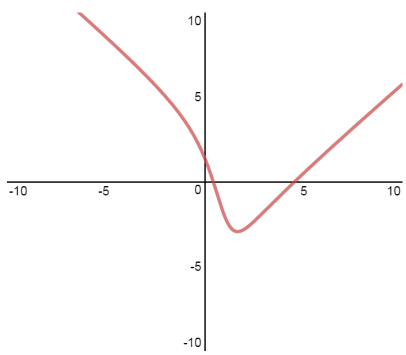
$$f(x) = \frac{\sqrt{x} + 5}{x^2 - 5} \qquad f(x) = (\sqrt{x} + 3)(x^4 - 2x) + \frac{3}{x^2}$$

The graphs of algebraic functions come in many unique shapes. A few examples are shown below.

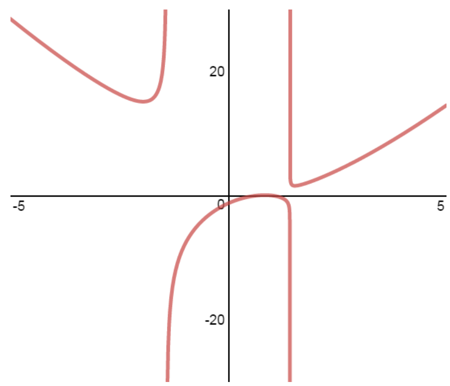
$$f(x) = \frac{\sqrt{x} - 5}{x^2 + 7}$$



$$f(x) = \frac{x^2 - 5x + 2}{\sqrt{x^2 - 2x + 2}}$$



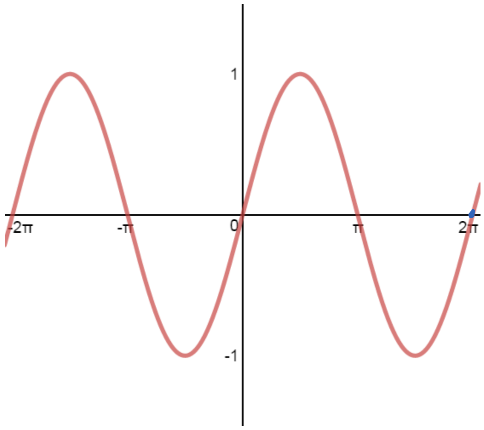
$$f(x) = \frac{3x^2 - 5x + 2}{\sqrt[3]{2x^2 - 4}}$$



Trigonometric Functions [VID_7851]

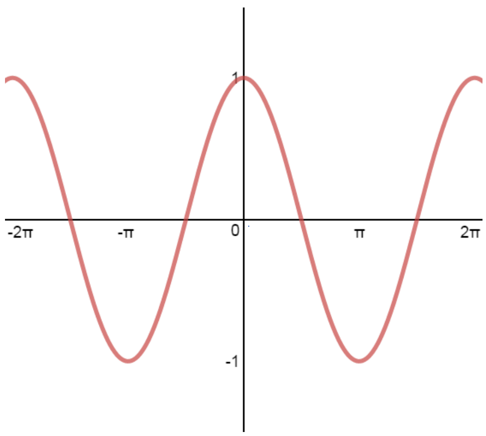
The main trigonometric functions are $\sin(x)$, $\cos(x)$, and $\tan(x)$ and their respective reciprocals are $\csc(x)$, $\sec(x)$, and $\cot(x)$

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$



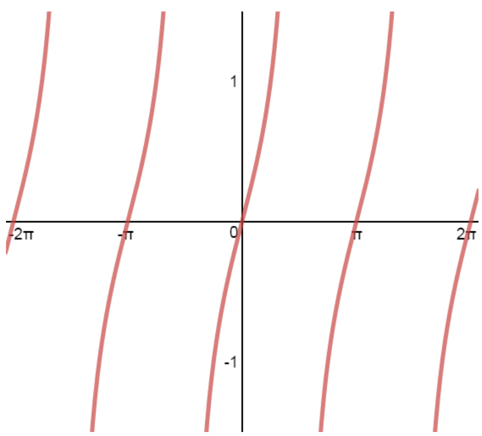
$f(x) = \sin(x)$

Domain: $x \in R$
Range: $-1 \le y \le 1$
Period: 2π



$f(x) = \cos(x)$

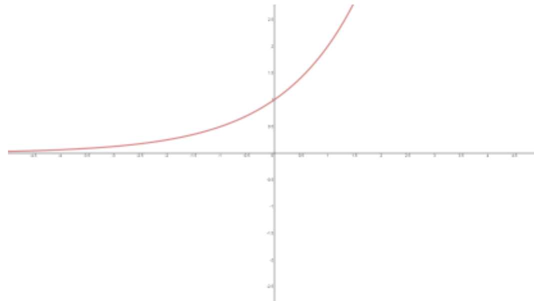
Domain: $x \in R$
Range: $-1 \le y \le 1$
Period: 2π



$f(x) = \tan(x)$

Domain: $x \neq \cdots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$
Range: $y \in R$
Period: π

What is an exponential function?



$f(x) = a^x$

a is called the base and $a > 0, \quad a \neq 1$
x is called the exponent

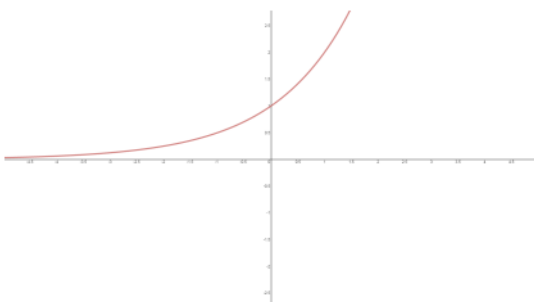
Key Features

- y-intercept is always 1, no matter what the base is
- There is no x-intercept, there is a horizontal asymptote at $y = 0$
- The domain is $x \in R$
- The range is $y > 0$

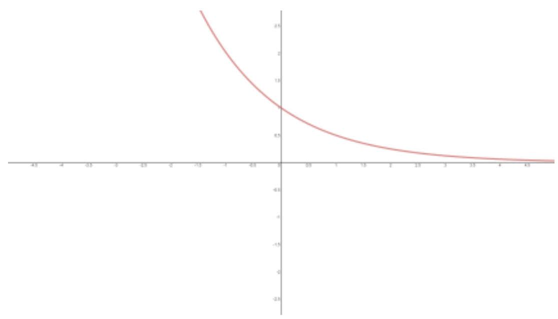
Note: e is a special math number which is equal to 2.718...
(similar to how $\pi = 3.14$....)

Therefore $y = e^x$ is a exponential function with base e

$a > 1$



$0 < a < 1$



$a > 1$

- As x values get more positive (∞), the y-value also get's larger and larger
- As x values get more negative ($-\infty$), the y-value approaches 0, but will never become 0

$0 < a < 1$

- As x values get more negative (-∞), the y-value also get's larger and larger
- As x values get more positive(∞), the y-value approaches 0, but will never become 0

What Are Logarithms?

In simple terms, the logarithm is equal to the exponent to which a number (the base) must be raised in order to produce a given number. Another way to think of logarithms, is that it is the inverse of exponential functions. Much like squaring a term and taking the square root are inverse operations.

Logarithmic Form	**Exponential Form**	
$y = \log_a x$	$x = a^y$	$3 = log_2 8 \leftrightarrow 8 = 2^3$

The logarithmic equation would be read as 'log of x to the base a' OR 'log base a of x'

For both logarithms and exponents there is a base. Notice how the base is the same number when converting from logarithmic to exponential form and vice versa.

Note the following features of the basic logarithmic graph $f(x) = \log_{10} x$

- x intercept is 1
- There is no y intercept
- There is a vertical asymptote at $x = 0$
- As x approaches 0, $f(x) = -\infty$
- As x approaches ∞, $f(x) = \infty$
- Domain is $x > 0$
- Range is $y \in R$