MATH1013.com

We Make Math Easy.

Chapter 1.1

Tutorial Length
2 Hrs 30 Mins

Homework ~ Tutorials ~ Past Tests

Important

Math 1013 is a HUGE course. Many students fear the course, but you don't need to, you've got us! The keys to success are to practice as many types of problems as possible and not to fall behind. Each chapter builds on the concepts of a previous chapter so it's crucial to understand the material from one chapter before moving on.

This is where MATH1013.COM comes in. We have developed extensive tutorial videos for each section that will give you a quick overview of the theory before we jump in to examples. Our goal is to make things as simple as possible. We will go through MANY examples in order to ensure you understand the concept. We want to show that one concept can be tested in multiple different ways. By making your way through all the questions, you will see different variations and learn new techniques that will make MATH 1013 a breeze. We'll show you shortcuts, easy tricks to remember, and even go through past test questions.

In short, if you're reading this, you're already on the right path. Your success is our success and we wish you the best with this course.

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Helpful Tips

- Each question has a 4 digit video ID code. If you only want to watch a specific example, just search for the 4 digit code in the playlists.
- Some sections are very long. Consider breaking it down into smaller periods of time (1 hour chunks) in order to efficiently absorb the information.
- When going through tutorial videos, if you are having a particularly difficult time with a question, skip it and come back later. Sometimes the brain just needs a bit of a break!
- Keep all of your MATH1013.com booklets in a binder. This way when it's time to do a final review for a test, you can quickly go through the material. Try to circle or highlight key points. These items will stand out when you begin reviewing.
- If you've purchased access to past tests, don't go through those questions until you feel you've learned all the material. Then go through as many past tests as possible in preparation for your actual test. Once you go through the solutions, you will see where you still have issues and what you still need to review.

Policy Reminder:

While sharing is caring, any user accounts found to be shared between students will be terminated with no refunds. Additionally, access to all premium content will expire after the final exam. Please see the account terms and conditions for more details.

Contact

Questions, Concerns, Comments? info@math1013.com
Please note we are unable to offer tutoring assistance over e-mail.

Functions

What Is A Function?

A function is simply an equation that assigns **one output value for each unique input value.** In other words, for any given x-value, the equation only creates a single y-value.

Example of equations that are functions:

$$y = 4x + 1$$

$$y = \frac{5x - 3}{x^2 + 1}$$

$$y = \sqrt{2x + 3}$$

A common notation for functions is $f(x)$ instead of y. For example, $f(x) = 4x + 1$

x is considered the **independent variable**, this is what we consider the input value.
y is considered the **dependent variable**, this is what we consider the output value, and it is dependent on the input value x.

The function is not always in terms of x and y.

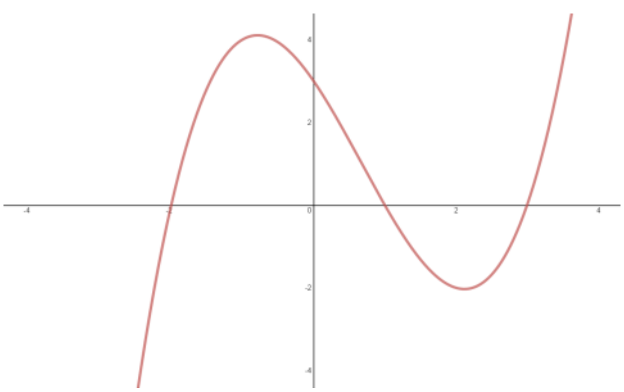
$A = \pi r^2$ $\qquad$ $A(r) = \pi r^2$

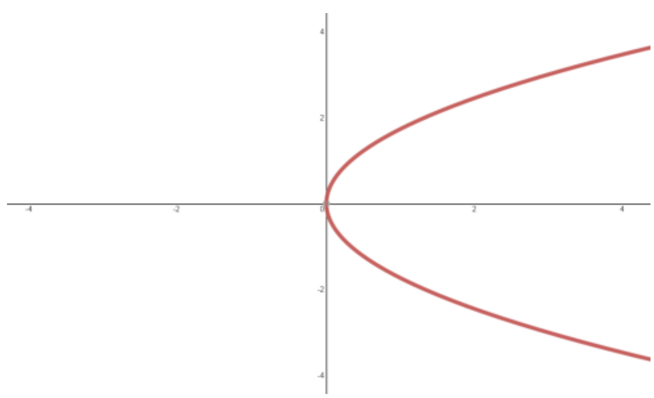
$s = 5t^2 + 2t - 5$ $\qquad$ $s(t) = 5t^2 + 2t - 5$

All valid input values make up what is called the **domain** of the function.
All valid output values make up what is called the **range** of the function.

The Vertical Line Test

When presented with a graph, if you can draw a vertical line at all points and it only ever intersects the graph at **only one point**, it is a function. Looking at it another way, if you draw a vertical line at any point and it intersects the graph at **more than one point**, it is NOT a function.





The vertical line tests works because if for any given x-value there is more than one y-value, then the graph CANNOT be a function.

How To Evaluate A Function?

To evaluate a function, simply replace the variable in the equation with what the function needs to be evaluated at.

Example (VID_4359) If $f(x) = 3x^2$ then determine the following:

$f(0)$

$f(-2)$

$f(a)$

$f(a+h)$

$f(\frac{2}{h})$

How To Evaluate A Function?

To evaluate a function, simply replace the variable in the equation with what the function needs to be evaluated at.

Example (VID_2807) If $g(x) = -2x^2 + 3x + 1$ then determine the following:

$g(2)$

$g(-1)$

$g(x + h)$

How To Evaluate A Function?

To evaluate a function, simply replace the variable in the equation with what the function needs to be evaluated at.

Example (VID_7815) If $h(t) = \frac{2t}{5+t}$ then determine the following:

$h(0)$

$h(5)$

$h(\frac{2}{a})$

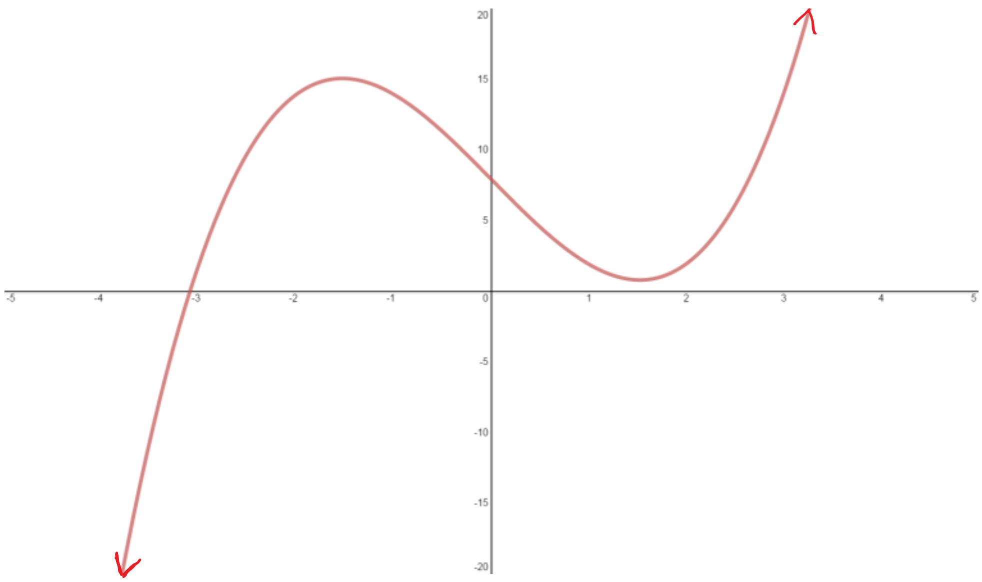
$h(\frac{1}{1+a})$

Recall…

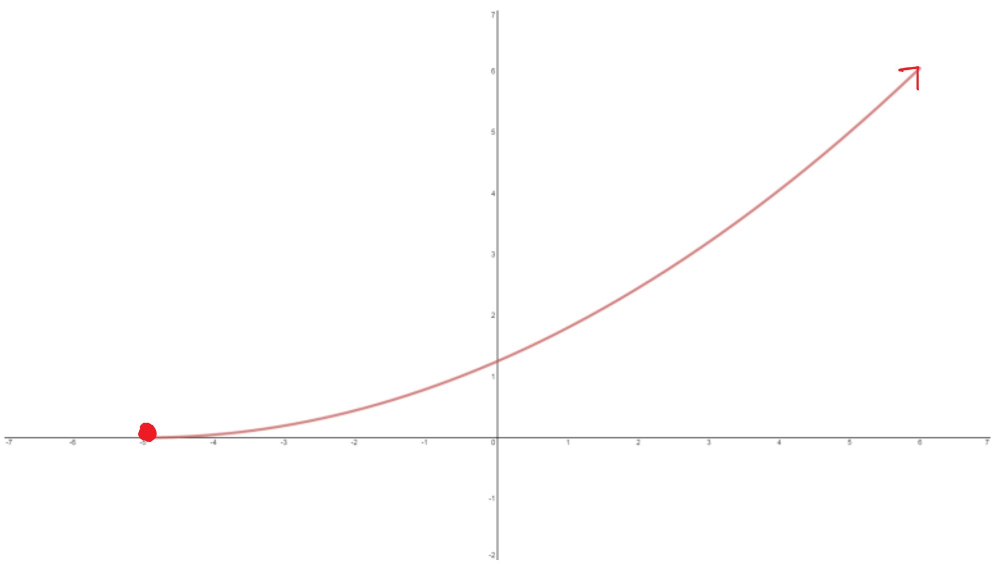
All valid input values make up what is called the **domain** of the function.
All valid output values make up what is called the **range** of the function.

Typically, the domain and range are written in set notation or interval notation.
Recall: On a graph, a solid filled in circle means the number is included (closed interval), and a hollow circle means the number is not included (open interval).

Example (VID_6902) Find the domain and range of the following function.



Example (VID_4499) Find the domain and range of the following function.



Recall…

All valid input values make up what is called the **domain** of the function.
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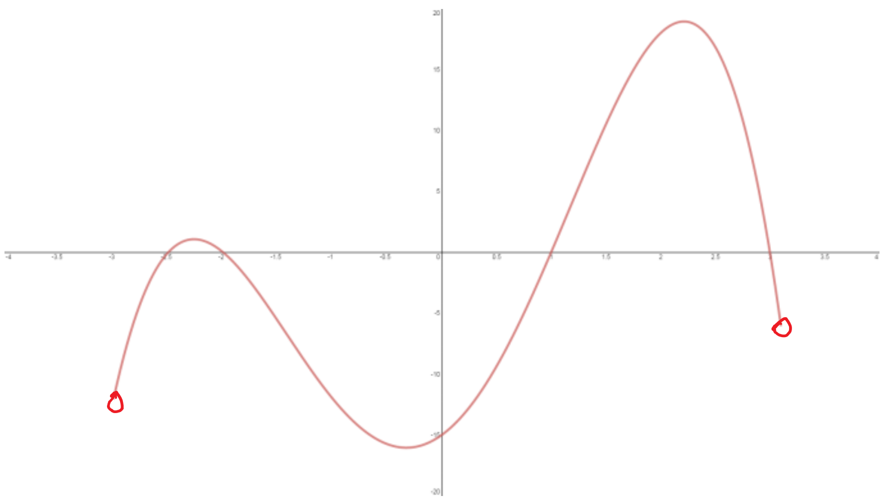
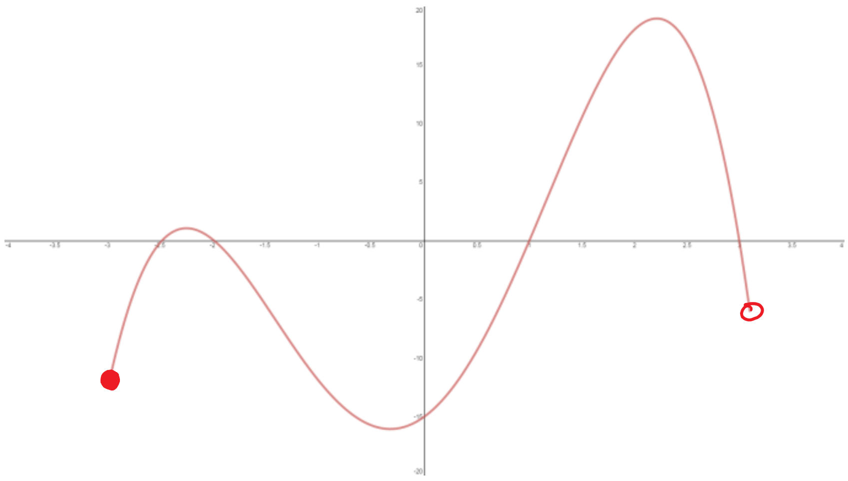
Typically, the domain and range are written in set notation or interval notation

Example (VID_2551) Find the domain and range of the following function.



Example (VID_2551) Find the domain and range of the following function.



Domain & Range of Polynomials

Recall…

All valid input values make up what is called the **domain** of the function.
All valid output values make up what is called the **range** of the function.

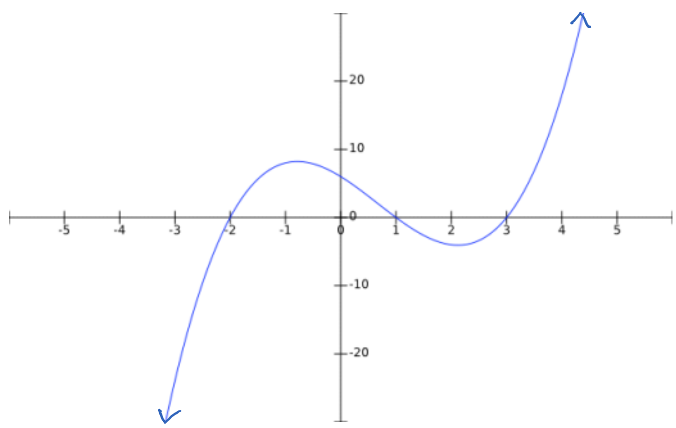
Typically, the domain and range are written in set notation or interval nation. The choice is yours

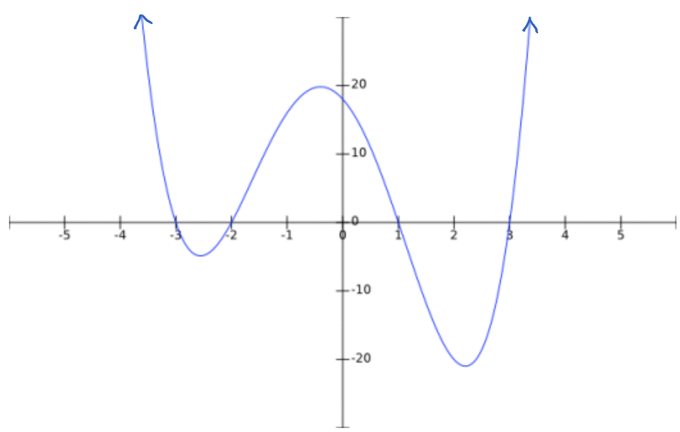
In general, a polynomial is represented in the following way:
$P(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots c_2 x^2 + c_1 x + c_0$

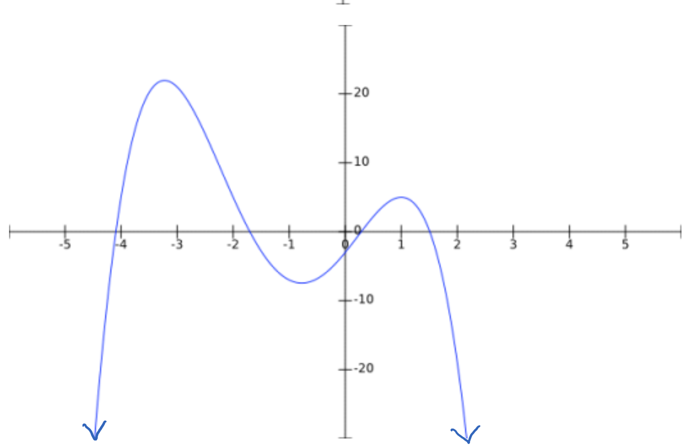
Domain of Polynomials

The domain of all polynomials is all real numbers ($x \in \mathbb{R}$)

This is because there is no value of x that would cause the equation to not be valid.
The range however can vary as seen in the plots of the following polynomials.







Finding The Domain Of Quotients

In general, the denominator of a quotient cannot equal to 0
(If you have a calculator nearby, plug in a number and divide by 0 and you will get an error)

$$\frac{f(x)}{g(x)} \quad domain\ is \quad g(x) \neq 0$$

Steps:

1) Set denominator not equal ($\neq$) to 0 and solve
2) Domain is all values except the values found in step 1 (write answer in interval or set notation)

Example (VID_1159) Find the domain of the following function:

$$f(x) = \frac{1}{x-1}$$

Example (VID_0936) Find the domain of the following function:

$$f(x) = \frac{3x}{x^2 - 6x - 16}$$

Example (VID_2084) Find the domain of the following function:

$$s(t) = \frac{t^2 + 2t + 1}{t^2 - 4}$$

Finding The Domain Of Quotients

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$$\frac{f(x)}{g(x)} \quad g(x) \neq 0$$

Steps:
1) Set denominator not equal ($\neq$) to 0 and solve
2) Domain is all values except the values found in step 1 (write answer in interval or set notation)

Example (VID_0916) Find the domain of the following function:

$$f(x) = \frac{5x + 4}{x^2 + 1}$$

Example (VID_3492) Find the domain of the following function:

$$f(x) = \frac{5x + 4}{x^3 + 1}$$

Finding The Domain Of Square Root Functions

In general, a number under a square root must be greater than or equal to 0.

$$for\ \sqrt{f(x)} \quad domain\ is\ \ f(x) \geq 0$$

(If you have a calculator nearby, plug in a negative number under the square root and you will get an error)

Steps:
1) Set function under the square root to be greater than or equal to 0 (≥ 0) and solve
2) Domain is solution to inequality in step 1 (write answer in interval or set notation)
   a. If it's a linear inequality, solve the inequality to get the solutions
   b. For all other types of inequalities, find the 'zeros' and use a chart

Example (VID_6428) Find the domain of the following function:
$f(x) = \sqrt{2x-1}$

Example (VID_1098) Find the domain of the following function:
$g(x) = \sqrt{5x+4}$

Example (VID_0972) Find the domain of the following function:
$h(t) = \sqrt{3-7t}$

Finding The Domain Of Square Root Functions

In general, a number under a square root must be greater than or equal to 0.

$$for\ \sqrt{f(x)} \quad domain\ is\ \ f(x) \geq 0$$

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Steps:
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    a. If it's a linear inequality, solve the inequality to get the solutions
    b. For all other types of inequalities, find the 'zeros' and use a chart

Example (VID_2318) Find the domain of the following function:

$$h(x) = \sqrt{x^2 - 3x - 10}$$

Example (VID_1242) Find the domain of the following function:

$$f(x) = \sqrt{9 - x^2}$$

Finding The Domain Of Square Root Functions

In general, a number under a square root must be greater than or equal to 0.

$$for\ \sqrt{f(x)} \quad domain\ is\ \ f(x) \geq 0$$

Quadratic Shortcut SQUARE ROOT Domains

If $f(x)$ is a quadratic functions ($ax^2 + bx + c$), then the following shortcut may be used:

Step 1) Solve $f(x) = 0$ to find the roots of the equation.
Note: The 'roots' of an equation is what x-values make the equation equal to 0.
Step 2) Solve the question based on the following:

a. If a is positive in ax^2, then the solution is OUTSIDE and INCLUDING the roots
b. If a is negative in ax^2, then the solution is INSIDE and INCLUDING the roots

Example (VID_3358) Find the domain of the following function:

$$h(x) = \sqrt{x^2 - 3x - 10}$$

Example (VID_5257) Find the domain of the following function:

$$f(x) = \sqrt{9 - x^2}$$

Finding The Domain Of Square Root Functions

In general, a number under a square root must be greater than or equal to 0.

$$for\ \sqrt{f(x)} \quad domain\ is\ \ f(x) \geq 0$$

Quadratic Shortcut For SQUARE ROOT Domains

If $f(x)$ is a quadratic functions ($ax^2 + bx + c$), then the following shortcut may be used:

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a. If a is positive in ax^2, then the solution is OUTSIDE and INCLUDING the roots
b. If a is negative in ax^2, then the solution is INSIDE and INCLUDING the roots

Example (VID_6930) Find the domain of the following function:

$$h(x) = \sqrt{2x^2 - 5x - 3}$$

Advanced Topic

Finding The Domain - Root Functions

In general, a number under an EVEN root must be greater than or equal to 0.
There are no such restrictions for an odd root.

$$for\ \sqrt[n]{f(x)} \quad domain\ is\ \ f(x) \geq 0\ \ if\ n\ is\ EVEN$$
$$for\ \sqrt[n]{f(x)} \quad no\ restriction\ if\ n\ is\ ODD$$

Remember, a square root is when $n = 2$ (the 2 is not normally shown on the root sign)

Example (VID_ 0348) Find the domain of the following function:
$f(x) = \sqrt{x-1}$

Example (VID_ 0348) Find the domain of the following function:
$g(x) = \sqrt[3]{x-1}$

Example (VID_5649) Find the domain of the following function:
$h(x) = \sqrt[3]{x^3 + 3x + 1}$

Finding The Domain - Combining Square Roots and Quotients

Recall...

$$for\ \sqrt{f(x)} \quad domain\ is\ \ f(x) \geq 0$$

$$for\ \frac{f(x)}{g(x)} \quad domain\ is\ \ g(x) \neq 0$$

Steps:

1) Find all domain restrictions of the quotient
2) Find all domain restrictions under the root
3) Find the greatest common domains and write answer in interval or set notation.

Example (VID_7072) Find the domain of the following function:

$$f(x) = \frac{1}{\sqrt{x-1}}$$

Example (VID_9549) Find the largest domain of the following function:

$$g(x) = \frac{5x^2 + 2x}{\sqrt{4-x}}$$

Example (VID_0581) Find the largest domain of the following function:

$$f(x) = \frac{\sqrt{4 - x^2}}{x}$$

Example (VID_4967) Find the largest domain of the following function:

$$f(x) = \frac{\sqrt{x^2 - 9}}{x - 1}$$

Example (VID_8995) Find the largest domain of the following function:

$$g(x) = \frac{-2x}{\sqrt{x^2 + 9}}$$

Example (VID_6725) Find the largest domain of the following function:

$$s(t) = \sqrt{\frac{t+3}{t-2}}$$

Example (VID_7315) Find the largest domain of the following function:

$$h(x) = \sqrt{\frac{4-3x}{x-2}}$$

Example (VID_3724) Find the largest domain of the following function:
$h(x) = \sqrt{x+4} - \sqrt{2-x}$

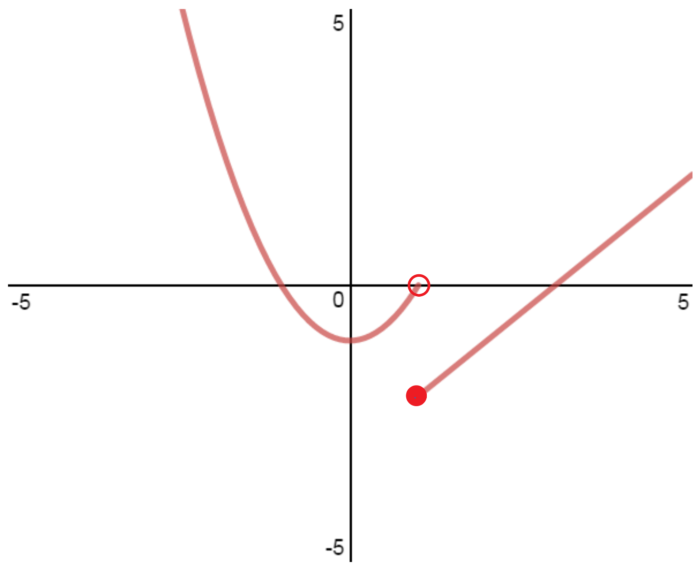
Example (VID_8797) Find the largest domain of the following function:
$g(x) = \sqrt{x^2-4} + \sqrt{2x+1}$

Example (VID_7019) Find the largest domain of the following function:
$$f(x) = \frac{1}{x-1} + \frac{1}{x+2}$$

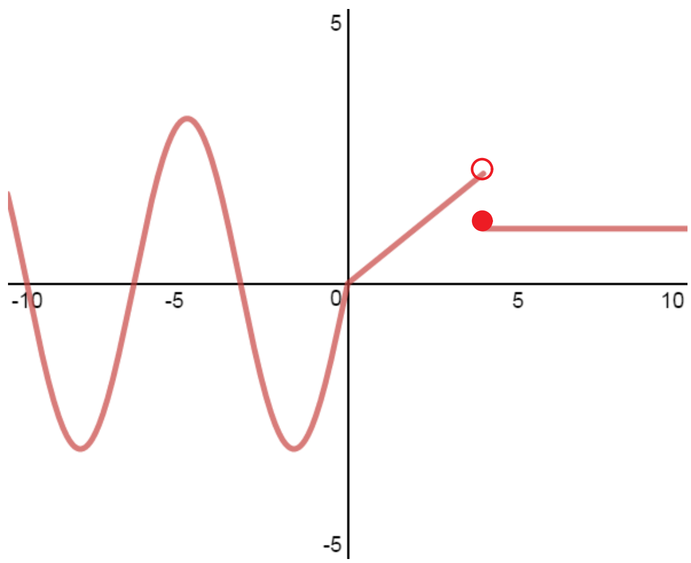
Piecewise Defined Functions

A piecewise function is a function that is broken up into different equations (formulas) for different intervals on the domain.

$$f(x) = \begin{cases} x^2 - 1 & if\ x < 1 \\ x - 3 & if\ x \geq 1 \end{cases}$$



$$f(x) = \begin{cases} 3\sin(x) & if\ x < 0 \\ 0.5x & if\ 0 \leq x < 3 \\ 1 & if\ x \geq 3 \end{cases}$$



Domain & Range of Piecewise Defined Functions

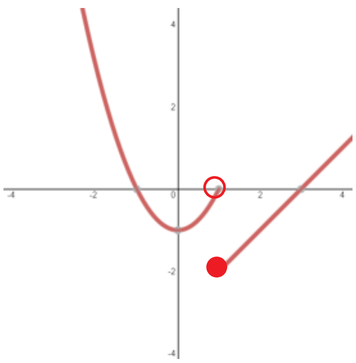
The domain of a piecewise function can be determined by analyzing each equation individually with the interval it is to be used in. Determine if each interval would require an additional restriction. Once each equations interval restrictions are analyzed, we can combine the intervals to find the overall domain of the piecewise defined function.

Range can be found by determining the range for each equation individually (within its given interval of valid input values), and then combining the individual ranges to find the overall range.

Example (VID_4865)

Find the domain and range of the following function.

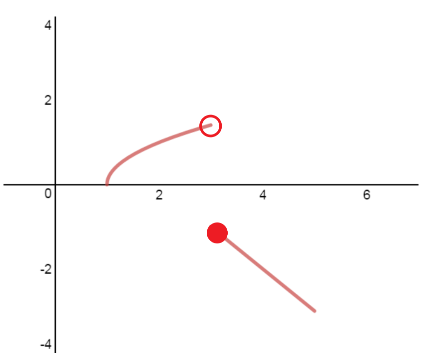
$$f(x) = \begin{cases} x^2 - 1 & if\ x < 1 \\ x - 3 & if\ x \geq 1 \end{cases}$$



Example (VID_4761)

Find the domain and range of the following function

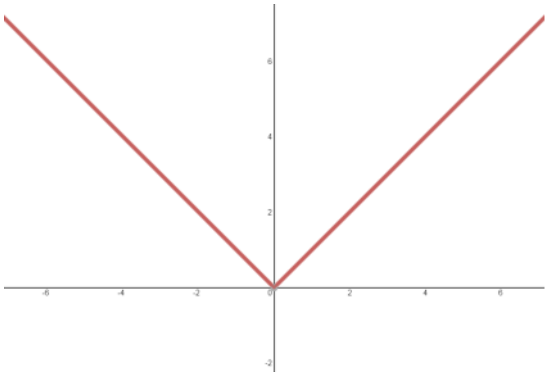
$$f(x) = \begin{cases} \sqrt{x - 1} & if\ x < 3 \\ -x + 2 & if\ 3 \leq x \leq 5 \end{cases}$$



Absolute Value

The absolute value function is a piecewise defined function.
Two straight lines, | |, are used to denote the absolute value function.

$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$



Examples [VID_4271]

Solve the following

$|-5| =$

$|7| =$

$|-3+8| =$

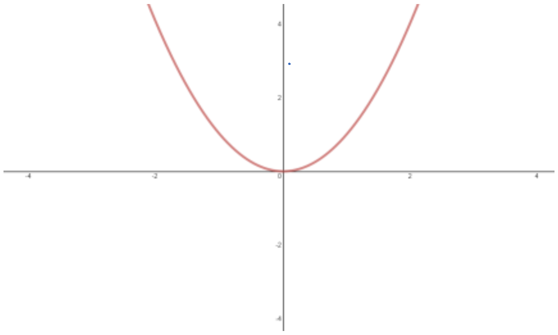
$|-6+4| =$

Note: The output of an absolute function can only be 0 or a positive number!

$|x| = -7$
There is no value of x that would cause this to be true!

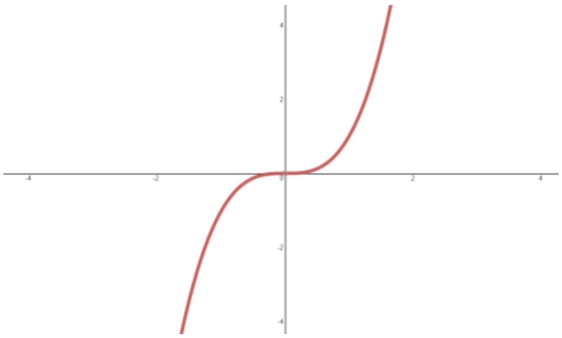
What Is Symmetry

There are 2 types of symmetry we are concerned with, odd symmetry and even symmetry. Sometimes these are referred to as odd functions or even functions.



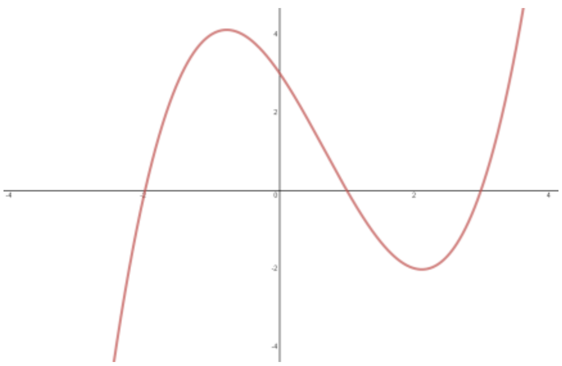
Even Symmetry
If the following is true for a function, then it shows even symmetry and it is an even function. These functions are symmetric about the y-axis

$f(-x) = f(x)$



Odd Symmetry
If the following is true for a function, then it is shows odd symmetry and it is an odd function. These functions are symmetric about the origin. 180* ROTATION

$f(-x) = -f(x)$
Or
$-f(-x) = f(x)$



No Symmetry

A function can have even symmetry OR odd symmetry OR no symmetry. It can't be both even and odd at the same time!

Equation Examples

Usually, you are not given a graph to determine symmetry. Instead, you have to figure out if an equation has even/odd/no symmetry using the following formulas.

Even Symmetry
$f(-x) = f(x)$

Odd Symmetry
$f(-x) = -f(x)$

Steps for determining symmetry:

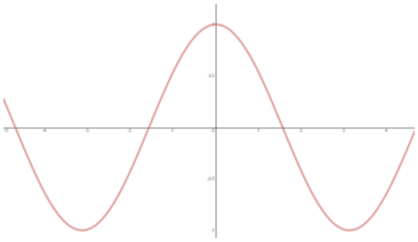
1) Determine your function $f(x)$
2) Find $f(-x)$ by replacing every x in $f(x)$ with $-x$
   a. Simplify and compare to $f(x)$. If it's the same, it's EVEN, otherwise go on to step 3.
3) Find $-f(x)$ by multiplying $f(x)$ by -1
   a. Simplify and compare to $f(-x)$ from step 2. If it's the same, it's ODD, otherwise go on to step 4.
4) If you made it to step 4, it means there is NO symmetry present. It is not even or odd.

Example (VID_3846) Determine if $f(x) = 3x^2 - 1$ has even, odd, or no symmetry.

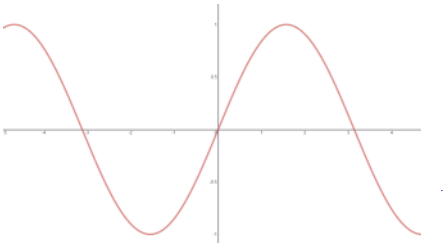
Example (VID_9074) Determine if $f(x) = \frac{x}{\sqrt{x^2+1}}$ is even, odd, or neither.

Example (VID_6433) Determine if $f(x) = x^3 + 2x + 1$ is even, odd, or neither.

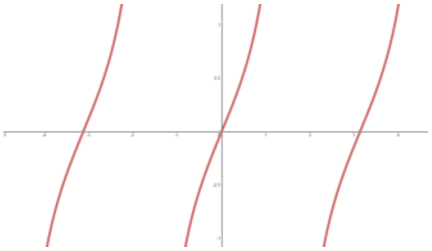
Graphical Examples (VID_2850)



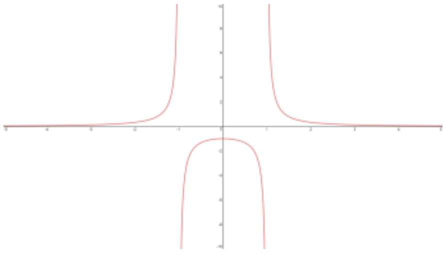
This is a plot of $f(x) = \cos(x)$
Does it show any kind of symmetry, if so, is it even or odd?



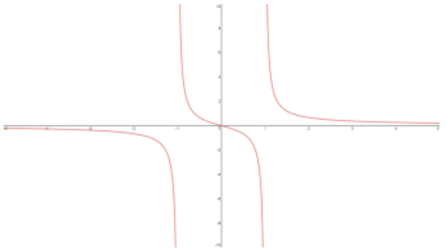
This is a plot of $f(x) = \sin(x)$
Does it show any kind of symmetry, if so, is it even or odd?



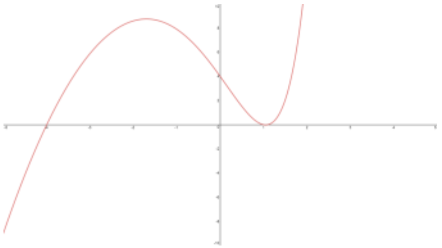
This is a plot of $f(x) = \tan(\mathrm{x})$
Does it show any kind of symmetry, if so, is it even or odd?



This is a plot of $f(x) = \frac{x}{x^3 - x}$
Does it show any kind of symmetry, if so, is it even or odd?



This is a plot of $f(x) = \frac{x^2}{x^3 - x}$
Does it show any kind of symmetry, if so, is it even or odd?



This is a plot of $f(x) = (e^x - 3)(x - 1)(0.5x - 2)$
Does it show any kind of symmetry, if so, is it even or odd?

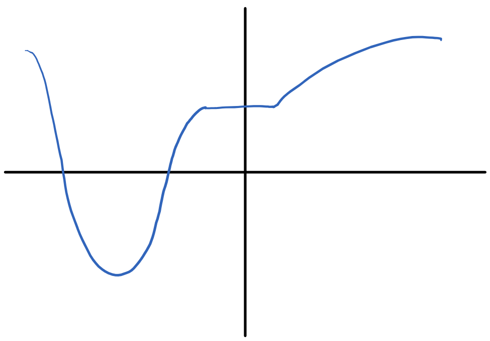
Increasing & Decreasing Functions

Increasing Functions

A function is increasing on a given interval when
$f(x_1) < f(x_2)$ where $x_1 < x_2$
(Each successive y-value is larger than the previous)

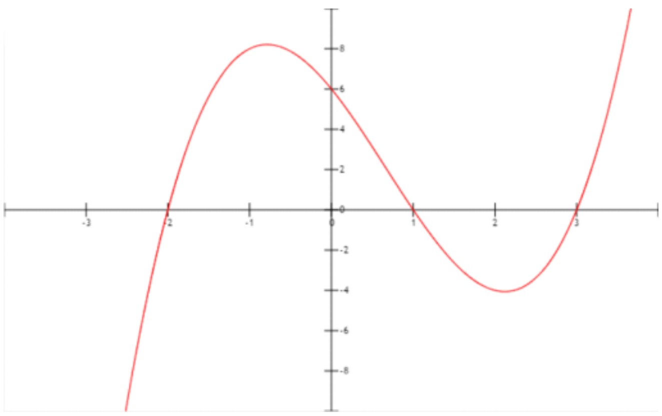
Decreasing Functions

A function is decreasing on a given interval when
$f(x_1) > f(x_2)$ where $x_1 < x_2$
(Each successive y-value is smaller than the previous)



Example (VID_0170)

Find the intervals where the following function is increasing and decreasing.



Example (VID_5855)

Find the intervals where the following function is increasing and decreasing.